Roll No. ....

DD-460

M. A./M. Sc. (Second Semester) EXAMINATION, May-June, 2020

MATHEMATICS

Paper Second

(Real Analysis—II)

Time : Three Hours

Maximum Marks : 80

Note : Solve any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) State and prove the Fundamental theorem of calculus.

(b) Let $Y : [a, b] \to R^k$ be a curve. If $c \in (a,b)$, then prove that :

$$\wedge y(a,b) = \wedge y(a,c) + \wedge y(c,b)$$

(c) Explain the existence of Riemann-Stieltjes integral.

Unit—II

2. (a) Explain Borel and Lebesgue measurability.

(b) Prove that the outer measure of an interval is its length.

(c) Define Lebesgue integral. Evaluate the Lebesgue integral of the function $f: [0\ 1] \rightarrow R$ defined by :

$$f(x) = \begin{cases} \dfrac{1}{x} & \text{if } 0 \le x \le 1 \\ 13 & \text{if } x = 0 \end{cases}$$

Show that f is not Lebesgue integrable on [0, 1].

Unit—III

3. (a) Explain extension of a measure. If $A \in Q$, then prove that :

$$\mu^*(A) = \mu(A)$$

(b) Define outer measure. Let the sets $E_1, E_2, ..., E_n$ be disjoint and measurable. Then prove that :

$$\mu^*\left[A \cap \left\{\bigcup_{i=1}^{n} E_i\right\}\right] = \sum_{i=1}^{n} \mu^*(A \cap E_i)$$

(c) Explain Reimann and Lebesgue integrals.

Unit—IV

4. (a) Let $f : R \rightarrow R$ be defined by :

$$f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then calculate the four derivatives of f. Is f differentiable at $x = 0$.

(b) State and prove Lebesgue differentiation theorem.

(c) Explain the four derivatives (Dini derivatives). Give an example of differentiable function (via Dini derivatives).

Unit—V

5. (a) Define a function of bounded variation. Show that a monotonic function on $[a, b]$ has finite variation.

(b) Give an example of L^p-space. State and prove Holder inequality for $0 < p < 1$.

(c) State and prove the Minkowski's inequality.