Roll No. .....

DD–2807

M. A./M. Sc. (Final) EXAMINATION, 2020

MATHEMATICS

(Compulsory)

Paper Second

(Partial Differential Equations and Mechanics)

Time : Three Hours

Maximum Marks : 100

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Derive fundamental solution of Laplace's equation. State and prove the symmetry of Green's function for Laplace's equation.

   (b) State and prove the uniqueness and backward uniqueness for neat equation.

   (c) Define wave equation with physical interpretation. Derive solution by spherical means for $n = 1$.

Unit—II

2. (a) Derive local existence theorem for non-linear partial differential equation.

   (b) State and prove the Lay-Oleinik formula.

   (c) Derive Asymptotics in L′-norm.

Unit—III

3. (a) Derive Lagrange's equations of first kind.

(b) Find the path which allows a particle to accomplish the transit in the least possible time, when it moves under the influence of gravity only, starting at rest from some fixed point to some other lower point (fixed).

(c) Derive Euler's equation for one dependent variable.

Unit—IV

4. (a) Derive Hamilton's principle from Newton's equations and Lagrange's equation.

(b) Derive Lee-Hwa-Chung theorem.

(c) Derive Invariance of Poisson brackets under Canonical transformations.

Unit—V

5. (a) If AB is a straight rod of small cross-section k and uniform density ρ, then to determine its potential at any external point P.

(b) Two thin straight uniform rods AB and CD are pivoted together at their middle points. Show that the attraction between them reduces to a couple of moment :

$$2\,Y\,mm'\,(AB-BC)\operatorname{cosec}\alpha$$

where m and m' are their line densities and α is the angle between them.

(c) Derive Poisson's equation for Cartesian co-ordinates.